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The conformal radius as a function and its gradient image

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Abstract

Let Ω be a domain in $\mathbb{C}$ with three or more boundary points in $\mathbb{C}$ and R(w, Ω) the conformal, resp. hyperbolic radius of Ω at the point $w \in \Omega \setminus \{\infty\}$. We give a unified proof and some generalizations of a number of known theorems that are concerned with the geometry of the surface $S\Omega = \{(w,h) \mid w \in \Omega, h = R(w, \Omega)\}$ in the case that the Jacobian of $\nabla R(w, \Omega)$, the gradient of R, is nonnegative on Ω. We discuss the function $\nabla R(w, \Omega)$ in some detail, since it plays a central role in our considerations. In particular, we prove that $\nabla R(w, \Omega)$ is a diffeomorphism of Ω for four different types of domains.